The northern regions of Italy are the heartlands of the Italian textile industry, providing textile products for many great Italian (and non-Italian) fashion houses. Most of today's companies were founded at the turn of the century and have grown and thrived despite a series of twentieth century catastrophes, including the Great Depression and World War II. Sales in the entire Italian textile and apparel industry were about $80 billion in 2007. The Italian textile-fashion industry employs over 400,000 people.

Filatoi Riuniti is a family-owned spinning mill located in northwest Italy. They produce cotton yarn, which is one of the raw materials that is used to produce the fabrics that are then cut and sewn by Italian stylists into the worldwide famous Italian clothes.

Demand for Filatoi Riuniti's production is strong, but their spinning machine capacity is insufficient to meet its production orders. They decided to outsource part of the spinning production to six local family-owned spinning mills: Ambrosi, Bresciani, Castri, De Blasi, Estensi, and Giuliani. The local mills charge higher prices for spinning finer yarns, so Filatoi Riuniti has decided to spin as much as possible of the finer yarns entirely in-house and to outsource only the spinning of low-end (coarser) yarns. Last month, they faced a total demand of 104,500 kg of cotton and they outsourced 32,000 kg of the low-end sizes.

Filatoi Riuniti hired a consulting firm to see if the outsourcing strategies to the six local mills could be improved. After analyzing the data, they immediately saw the potential for very large savings through more optimal outsourcing strategies. In this problem, we'll improve the outsourcing strategy of Filatoi Riuniti using linear optimization. (Note: This problem is based off of a real case, but the names have been changed for privacy reasons.)

Problem 1- Formulating the Optimization Problem

Filatoi Riuniti produces four different sizes of yarn: extra fine, fine, medium, and coarse. Their current strategy is to produce all of the fine and extra fine yarn in-house, outsource some of the medium yarn to the lowest-price mill, and outsource all of the coarse yarn to the other mills. To try to improve this strategy, let's set up an optimization problem.

The decision variables are the amount of yarn of each size that each company should be assigned to produce. There are four sizes, and seven companies (including Filatoi Riuniti). These decision variables are set up in the spreadsheet. Note that the decision variables should be non-negative - none of the mills can produce a negative amount of yarn!

The objective is to minimize costs. The prices charged by the six local mills for production, as well as the production cost at Filatoi Riuniti, are provided in the spreadsheet. Additionally, the yarn that is spun by the six local mills needs to be transported. The transportation costs per kg of yarn are also provided in the spreadsheet.

Our model has two types of constraints: capacity constraints and demand constraints. The spreadsheet contains a table showing the production capacity and production rate per product for each of the mills (including Filatoi Riuniti). For example, at the Bresciani mill, it takes 0.70 hours to produce one kg of extra fine yarn, and there are a total of 3,000 machines hours per month available at this mill. There is also a table that estimates the demand for the four yarn sizes in the current month. We should produce at least the demand for each yarn type.

Formulate and solve this problem in Excel. The objective should minimize the sum of the production and transportation costs. You should have the following constraints: non-negative decision variables, a capacity constraint for each mill, and a demand constraint for each type of yarn. Remember that Ambrosi and De Blasi can't produce extra fine yarn, so you should also account for this.

What is the objective value of the solution?

Answer: 1382544.334

How many kg of medium yarn is outsourced?

Answer: 2040kg + 7143kg = 9183kg.

How many kg of fine yarn is outsourced?

Answer: 6250kg

At which mills does Filatoi use the maximum capacity available?

Answer: All except De Blasi

Problem 2 - Sensitivity Analysis

In Problems 2 and 3, we'll be checking how sensitive our model is to the data and constraints. If you change the constraints or data values to answer any of the questions, make sure to change them back to the original values before continuing on to the next problem.

Filatoi Riuniti should obviously consider increasing its spinning machine capacity. They could slightly expand the production capacity of the existing machines by renting an upgrade. This would increase their spinning production capacity by 600 hours/month. The monthly rental cost is $1,500/month. Would you recommend that they rent the upgrade?

Answer: We increase the capacity constraint for Filatoi Riuniti from 38000 to 38600 and see the objective changes from 1382544.33 to 1381273.75, a decrease of $1270.58, not enough to justify the 1500$ cost. Therefore no, they should not rent the upgrade.

Alternatively, Filatoi Riuniti could increase its spinning machine capacity by renting another spinning machine for the production of only medium size yarn, for a monthly rental cost of $3,000. The machine has a production capacity of 300 hours per month (the machine would run at the same rate of 0.425 hours/Kg). Suppose that the estimated production cost of running this machine is less than that for Filatoi Riuniti's existing machines and is estimated to be $5.70/Kg (as opposed to $11.40/Kg for their existing machines). Would you recommend that Filatoi Riuniti rent the machine?

Answer: We see that between the two companies to which medium yarn is being outsourced (De Blasi and Giuliani) De Blasi costs more per kg. Moreover, in the current solution De Blasi is producing 2040kg of medium yarn, which is more than the 300/0.425 = 706kg that this new machine would produce. Therefore, using the machine would take 706kg of production business away from De Blasi, saving Filatoi Riuiti (11.40-5.70)\*706kg = $4024, enough to justify the 3000$ cost.

Suppose that the shadow price of the medium yarn demand constraint is $12.30, with an allowable increase of 5388. If we increase the right hand side by more than 5388, the shadow price will change, and it is impossible to know how it changes without re-solving the model.

A new client is interested in purchasing up to 5,000 kg/month of medium size yarn. What is the minimum price per kg of yarn that Filatoi Riuniti should quote to this new client? (In answering this question, assume that Filatoi has not decided to increase its spinning machine capacity, and that Filatoi does not want to change the prices that they currently charge their existing clients.)

Answer: They should charge at least 12.30$/kg so as to cover the added costs that this new production will create.

Now suppose that the client wants to purchase 6,000 kg/month of medium size yarn. Now what is the minimum price per kg of yarn that Filatoi Riuniti should quote to this new client? (In answering this question, assume that Filatoi has not decided to increase its spinning machine capacity, and that Filatoi does not want to change the prices that they currently charge their existing clients.)

Answer: In Excel increase the demand constraint for medium yarn by 6000 and re-run the solver to find a new object of 1457237.88, an increase of 74693.55 from the original 1382544.33. This amounts to 74,693.55/6000 = 12.45$/kg, which is the minimum that should be charged.

Problem 3 - Data Estimates

If Filatoi Riuniti's production costs increase by 5%, by how much does the objective value of the solution change? You should adjust and re-optimize the model to answer this question.

Answer: In the Cost of Production table, for the row corresponding to Filatoi Riuniti, we multiply each value by 1.05 and re-run the solver to get a new objective of 1431468.82, an increase of 48924.49 over the original objective of 1382544.33.

If Filatoi Riuniti's production costs decrease by 5%, by how much does the objective value of the solution change (in absolute difference)? You should adjust and re-optimize the model to answer this question.

Answer: In the Cost of Production table, for the row corresponding to Filatoi Riuniti, we multiply each original value by 0.95 and re-run the solver to get a new objective of 1333619.84, a decrease of 48924.49 over the original objective of 1382544.33.

Now you think that the production capacity of one of your local mills, De Blasi, could vary within a 20% range of the estimated value. In the current solution, De Blasi produces 2040.125 kg of medium yarn, and none of the other types of yarn. If De Blasi's capacity is actually 20% higher or lower than the estimated value, will the solution change? HINT: Think about De Blasi's capacity constraint.

Answer: No. In the current solution only De Blasi is working only for 714 of up to 2600 hours possible. Even if we decrease this constraint by 20%, that still creates an upper bound of 2080 hours for De Blasi, still much higher than the 714 hours they are currently working.